Reply to Comment on "Optical-fiber-based Mueller optical coherence tomography"

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We reply to the comments by Park *et al.* [Opt. Lett. **29,** 2873 (2004)] about our previous Letter [Opt. Lett. **28,** 1206 (2003)].

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We do not agree with any of the comments made by Park *et al.*[1] about our Optics Letters paper.[2] The following replies appear in the order of their remarks.

In a previous publication[3] we analyzed in detail the limitation of using orthogonal incident polarization states for the determination of some special Jones matrices and indicated a solution based on nonorthogonal incident polarization states. We have already developed a technology that does not have this kind of limitation by using continuous source polarization modulation.[4] Most importantly, the algorithm reported in our Letter[2] is generic and is applicable no matter how a Jones matrix is acquired.

The algorithm used in Ref. 5 can recover tissue birefringence only when diattenuation is negligible. As diattenuation increases, we find that the error in the retardation that is calculated by their algorithm deteriorates (see Appendix A).

A Poincaré sphere is a graphic representation of the Mueller calculus. Rigorous analyses of their algorithm, including a proof of the equivalency of their algorithm to using $\mathbf{J}_{f2}^{-1}$ directly to treat $\mathbf{J}_{sf2}$, can be found in Appendix B. The claim itself made in Ref. 1 regarding diattenuation indicates that their algorithm cannot recover the diattenuation contrast and is not valid when diattenuation cannot be neglected.

Although in Ref. 1 it is claimed that the orientation of birefringence can be calculated by their algorithm, no theoretical validation or experimental evidence was provided to substantiate this claim.[5,6] In fact, based on polarimetry, we rigorously proved that the orientation of the birefringence computed by their algorithm is incorrect (see Appendix B), and the error of their algorithm has nothing to do with the offset that we first discovered in Ref. 2.

Since our objective was to report progress on fiber implementation in Mueller optical coherence tomography (OCT), the imaging speed was not a major concern in Ref. 2. Again, the algorithm reported in our Letter[2] is generic and is valid regardless of the imaging speed. Moreover, we later developed a system with the capability of imaging a Jones or Mueller matrix at a speed of 2–4 frames/s.[7]

Many papers on polarization-sensitive optical coherence tomography (PS-OCT) served as good references for our research. PS-OCT dates back to 1992,[8] and the original basic configuration remains the same in current PS-OCT. Mueller OCT—a branch of PS-OCT—merges rigorous conventional polarimetry and OCT synergistically.[9] As we know from polarimetry, a Jones or Stokes vector describes the polarization state of an optical beam, whereas a Jones or Mueller matrix describes the polarization properties of the material (tissue sample). Therefore the matrices and the vectors are not equivalent. Because it is the material that we are quantifying, it is the matrices that we are after. Previous work on PS-OCT measured only the Jones or the Stokes vectors. We are the first to quantify Mueller matrices using OCT.[9] We are also the first to discover the transpose symmetry in the round-trip Jones matrices measured by OCT.[10]

Appendix A: Error in the Retardation Calculated by the Algorithm in Ref. 5 as a Function of the Diattenuation

Here we illustrate the error in the retardation of the sample calculated with the algorithm in Ref. 5 as a function of the diattenuation of the sample. We tested the algorithm used in Refs. 5 and 6 for calculation of the phase retardation with different values of diattenuation, using a computer simulation. Diattenuation (D) is defined as $D = (q - r)/(q + r)$, where q and r are the intensity transmittances for the two orthogonal eigenpolarizations (eigenvalues of the Mueller matrix) of the diattenuator.[11] The sample has the following parameters: the round-trip retardation $\varphi_2 = 150°$, the orientation of the fast axis $\theta_2 = 45°$, $q = 0.9$, and r varies from 0.9 to 0.2. The birefringence in the sampling fiber is assumed, for simplicity, to be zero. The simulation result in Fig. 1 shows that when diattenuation is not negligible, even the phase

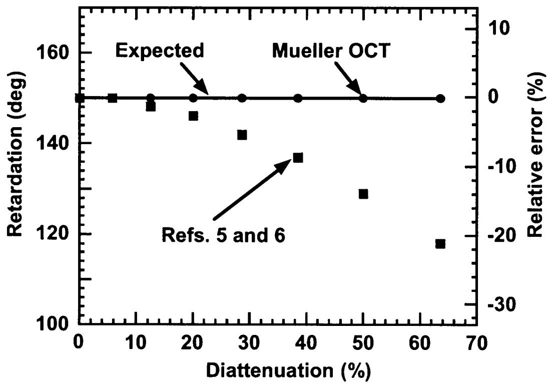

Fig. 1. Error in the retardation that is calculated by the algorithm in Refs. 5 and 6 as a function of the diattenuation.

retardation calculated with the algorithm used in Refs. 5 and 6 is incorrect.

Appendix B: Error in the Orientation of Birefringence Calculated by the Algorithm in Refs. 5 and 6

Here we use Jones calculus to analyze the error in the algorithm in Refs. 5 and 6 for the calculation of the orientation of birefringence in a sample, based on the fact that in OCT the polarization properties of the sample can be completely characterized by either a Mueller matrix or its equivalent Jones matrix.[12] In the following analysis we ignore the polarization properties of the detection arm in a fiber-based PS-OCT system without losing the generality of the conclusions. As illustrated in Fig. 2, the sampling fiber and the tissue segment can be characterized with Jones matrices $\mathbf{J}_{1f}$ and $\mathbf{J}_{2f}$ for the forward propagation and $\mathbf{J}_{1b}$ and $\mathbf{J}_{2b}$ for the backward propagation, respectively. $\mathbf{E}_{1i}$ and $\mathbf{E}_{2i}$ represent the input Jones vectors to the sampling fiber and the tissue segment, respectively. $\mathbf{E}_{1o}$ and $\mathbf{E}_{2o}$ represent the output round-trip Jones vectors measured by PS-OCT, where the former results from the round-trip transformation of the sampling fiber and the latter results from the round-trip sequential transformation of both the fiber and the tissue segment. These transformations between the vectors can be expressed as follows:

$$\mathbf{E}_{2i} = \mathbf{J}_{1f}\mathbf{E}_{1i}, \tag{B1}$$

$$\mathbf{E}_{1o} = \mathbf{J}_{1b}\mathbf{J}_{1f}\mathbf{E}_{1i}, \tag{B2}$$

$$\mathbf{E}_{2o} = \mathbf{J}_{1b}\mathbf{J}_{2b}\mathbf{J}_{2f}\mathbf{J}_{1f}\mathbf{E}_{1i}. \tag{B3}$$

We define a new matrix ($\mathbf{J}_{2r}$) to represent the round-trip Jones matrix of the tissue segment:

$$\mathbf{J}_{2r} = \mathbf{J}_{2b}\mathbf{J}_{2f}, \tag{B4}$$

which is the true matrix that we need to recover.

To eliminate the polarization effect of the sampling fiber, the ideal calibration process is to first calculate $\mathbf{J}_{1b}$ and $\mathbf{J}_{1f}$ based on Eq. (B2) and then use $\mathbf{J}_{1b}^{-1}$ and $\mathbf{J}_{1f}^{-1}$ to treat the combined Jones matrix in Eq. (B3) as we demonstrated in our Mueller matrix OCT.[2]

In Refs. 5 and 6 a rotation matrix was calculated to transform the Stokes vector corresponding to $\mathbf{E}_{1o}$ to the Stokes vector corresponding to $\mathbf{E}_{2o}$ in the Poincaré sphere—a graphic representation of the Mueller calculus—and was claimed to represent the polarization properties of the tissue segment. The Jones matrix that transforms $\mathbf{E}_{1o}$ to $\mathbf{E}_{2o}$ is equivalent to the rotation matrix above and can be derived by expressing $\mathbf{E}_{2o}$ with $\mathbf{E}_{1o}$.

Let us first look at the polarization properties of a single-mode optical fiber. Because of the varying orientations of birefringence along the fiber, an optical fiber is characterized in general as an elliptical retarder, and its one-way Jones matrix can be decomposed as[2]

$$\mathbf{J}_{1f} = \mathbf{J}_{1c}\mathbf{J}_{1l}, \tag{B5}$$

where $\mathbf{J}_{1l}$ and $\mathbf{J}_{1c}$ represent a linear and a circular retarder, respectively, with the following identities:

$$\mathbf{J}_{1l}^{T} = \mathbf{J}_{1l}, \tag{B6}$$

$$\mathbf{J}_{1c}^{T} = \mathbf{J}_{1c}^{-1}. \tag{B7}$$

Substituting $\mathbf{J}_{1b} = \mathbf{J}_{1f}^{T}$ (the Jones reversibility theorem) and Eqs. (B5)–(B7) into Eq. (B2), we obtain

$$\mathbf{E}_{1o} = \mathbf{J}_{1l}^{T}\mathbf{J}_{1l}\mathbf{E}_{1i}. \tag{B8}$$

By using Eq. (B6) we can invert Eq. (B8) to

$$\mathbf{E}_{1i} = \mathbf{J}_{1l}^{-1}\mathbf{J}_{1l}^{-1}\mathbf{E}_{1o}. \tag{B9}$$

We can then represent $\mathbf{E}_{2o}$ with $\mathbf{E}_{1o}$ by inserting Eqs. (B4), (B5), and (B9) into Eq. (B3):

$$\mathbf{E}_{2o} = \mathbf{J}_{1l}^{T}\mathbf{J}_{1c}^{T}\mathbf{J}_{2r}\mathbf{J}_{1c}\mathbf{J}_{1l}^{-1}\mathbf{E}_{1o}. \tag{B10}$$

We define a new matrix to represent the transformation in Eq. (B10):

$$\mathbf{J}_{\mathrm{Ref\ 5}} = \mathbf{J}_{1l}^{T}\mathbf{J}_{1c}^{T}\mathbf{J}_{2r}\mathbf{J}_{1c}\mathbf{J}_{1l}^{-1}. \tag{B11}$$

$\mathbf{J}_{\mathrm{Ref\ 5}}$ represents the rotation matrix in the algorithm of Refs. 5 and 6 in the Jones calculus and is not identical to the one that we are after, $\mathbf{J}_{2r}$. Both $\mathbf{J}_{\mathrm{Ref\ 5}}$ and $\mathbf{J}_{2r}$ can be converted to Mueller matrices if needed. The discrepancy between $\mathbf{J}_{\mathrm{Ref\ 5}}$ and $\mathbf{J}_{2r}$.

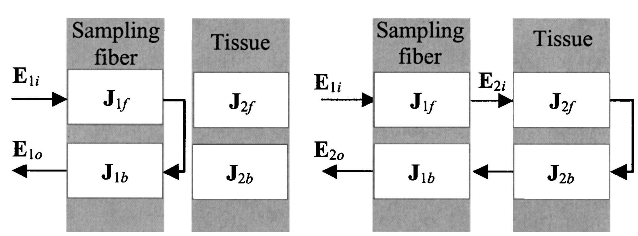

Fig. 2. Illustration of the polarization transformations in fiber-based PS-OCT.

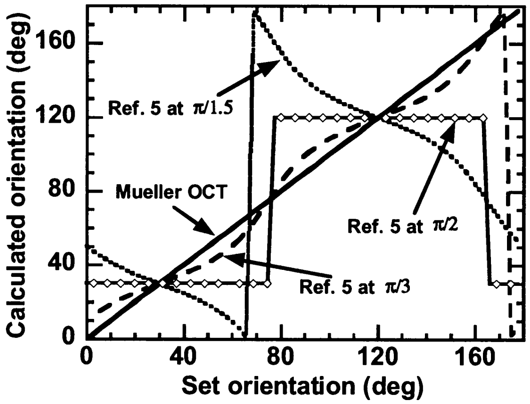

Fig. 3. Comparison between the orientations of the birefringence calculated by the algorithms in Refs. 2 (our Mueller OCT) and 5.

We use $\mathbf{J}_R(\varphi, \theta, \delta)$ to represent the Jones matrix of a general elliptical retarder,[3] where φ is the phase retardation, θ is an auxiliary angle, and δ is the phase difference between the two orthogonal components of the fast eigenvector. The orientation of the retarder (α) can be calculated from

$$\tan 2\alpha = \tan 2\theta \cos \delta \,. \tag{B12}$$

When diattenuation is negligible, we have proved that $\mathbf{J}_{2r}$ is reduced to a linear retarder $\mathbf{J}_R(\varphi_2, \theta_2, 0)$ because the circular retardation is canceled in the round-trip operation, as seen in Eq. (B8).[13] In this case, θ_2 represents directly the orientation of the fast axis. We then have the following relations:

$$\begin{aligned}
\mathbf{J}_{2r} &= \mathbf{R}(-\theta_2)\mathbf{J}_R(\varphi_2, 0, 0)\mathbf{R}(\theta_2)\,,\\
\mathbf{J}_{1l} &= \mathbf{R}(-\theta_1)\mathbf{J}_R(\varphi_1, 0, 0)\mathbf{R}(\theta_1)\,,\\
\mathbf{J}_{1l}^{-1} &= \mathbf{R}(-\theta_1)\mathbf{J}_R^*(\varphi_1, 0, 0)\mathbf{R}(\theta_1)\,,\\
\mathbf{J}_{1c} &= \mathbf{R}(\gamma_1)\,,\\
\mathbf{R}(\theta_1) &= \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix},
\end{aligned} \tag{B13}$$

where γ_1 is the rotation angle of the circular retarder. By using the identities

$$\begin{aligned}
\mathbf{R}(\theta_2 - \theta_1) &= \mathbf{R}(\theta_2)\mathbf{R}(-\theta_1)\,,\\
\mathbf{J}_R(\varphi_2, \theta_2, -\varphi_1) &= \mathbf{J}_R(\varphi_1, 0, 0)\mathbf{J}_R(\varphi_2, \theta_2, 0)\mathbf{J}_R^*(\varphi_1, 0, 0)\,,
\end{aligned} \tag{B14}$$

we can express $\mathbf{J}_{\mathrm{Ref}\,5}$ as

$$\mathbf{J}_{\mathrm{Ref}\,5} = \mathbf{R}(-\theta_1)\mathbf{J}_R(\varphi_2, \theta_2 + \gamma_1 - \theta_1, -\varphi_1)\mathbf{R}(\theta_1)\,, \tag{B15}$$

which shows that the phase retardation of $\mathbf{J}_{\mathrm{Ref}\,5}$ is equal to the phase retardation φ_2 of $\mathbf{J}_{2r}$. However, the orientation of birefringence calculated from $\mathbf{J}_{\mathrm{Ref}\,5}$ based on Eq. (B12) is

$$\alpha = 1/2 \tan^{-1}[\tan 2(\theta_2 + \gamma_1 - \theta_1)\cos\varphi_1] + \theta_1\,, \tag{B16}$$

which has a complicated nonlinear relationship with the true orientation θ_2 of the tissue segment calculated from $\mathbf{J}_{2r}$ unless φ_1 is zero, meaning that the sampling fiber is nonbirefringent and causes no distortion, which defeats the purpose of the calibration.

As an example, α was calculated from $\mathbf{J}_{\mathrm{Ref}\,5}$ with the following parameters: $\gamma_1 = 0$; $\varphi_1 = \pi/3$, $\pi/2$, $\pi/1.5$; $\theta_1 = \pi/6$; $\varphi_2 = \pi/3$; and θ_2 varies from 0 to π (Fig. 3). For comparison, α was also calculated with the algorithm of our Mueller-matrix OCT.[2] Obviously, the algorithm in Refs. 5 and 6 does not yield the correct orientation of the birefringence, whereas the algorithm in the Mueller-matrix OCT does.

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